We state the modern definition of circle, which we will use.

Definition.
A *circle* is the set of points in a plane which are equidistant to a given point. The given point is called the *center*, and the common distance from a point on the circle to the center is called the *radius*.

A *chord* of a circle is a line segment whose endpoints are on the circle.

The *diameter* of a circle is the length of a chord which passes through the center. This is clearly twice the radius.

A line or circle is *tangent* to a circle if they intersect in one point.

We sometimes call a line segment from the center to the circle a radius, and we sometimes call a chord passing through the center a diameter of the circle.

From the definition, it is apparent that a circle is completely determined by its center and radius; to circles are equal if and only if they have the same center and radius.

Euclid says that equal circles are those which have equal diameters; in modern language, we would say that two circles are congruent if they have the same diameter. Euclid says that if a line or a circle and a circle intersect in one point, they *touch*.

We rephrase Proposition III.1, to state what the proof shows.

Proposition 3. Center of a Circle
Given a circle and a chord in it, the center of the circle is the midpoint of the chord which is the perpendicular bisector of the given chord.

Corollary.
The center of a a circle lies on the perpendicular bisector of any chord. Thus, the center is the intersection of any two perpendicular bisectors.

Proposition 3. Chord Bisected by Diameter
Given a circle, a diameter, and a chord, the chord is perpendicular to the diameter if and only if they intersect in the midpoint of the chord.

We state Proposition III.4 as its contrapositive, which is equivalent and clearer.

Proposition 4. Chords Bisecting Each Other
Given a circle and two chords, if the chords bisect each other, they intersect at the center.

Propositions III.5, III.6, and III.10 are subsumed by the following clearer statement.

Propositions 5, 6, 10. Intersecting Circles
Distinct circles intersect in at most two points.

Proof. Let A, B, and C be points on a circle. The perpendicular bisectors of $\overline{AB}$ and $\overline{BC}$ intersect in a point E. This point is the center of the circle. The radius of the circle is EA. Any other circle containing points A, B, and C would have the same center and radius; thus, it would be the same circle. Therefore, distinct circles cannot have three points in common. □

Corollary.
Three noncollinear points determine a unique circle.

Proposition 9. Three Radii
Given a circle, a point inside, and three segments from the point to the circle, if the three segments are congruent, the point is the center.

Propositions 11, 12. Touching Circles
If two circles intersect in exactly one point, the line through the centers contains the point of intersection.

Proposition 14. Equal Bisectors
Given a circle, congruent chords are equally distant from the center, and chords equally distant from the center are congruent.

The following is, in Euclid, a "porism" of his Proposition 16. We only need the following statement.

Proposition 16. Perpendicular Lines are Tangent
If a line intersects a circle at right angles to a point intersection, they touch.

Construction 17. Tangent Construction
Given a circle and a point outside of it, it is possible to construct a line through the point which is tangent to the circle.

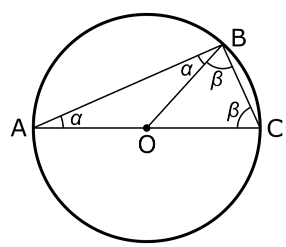
Proposition 18. Tangents are Perpendicular
Given a circle and a tangent line, the line is perpendicular to the radius from the center to the point of intersection.

Proposition 19. Center on Tangent's Perpendicular
Given a circle and a tangent line, the center of the circle lies on the line through the point of intersection and perpendicular to to given line.

What we call Thales' Theorem is actually a portion of Proposition 31 in Euclid, although it does not require any of Propositions 20 through 30 to prove. Indeed, all it requires is Book I, Proposition 5, the Base Angle Theorem. We place a modern proof here. Keep in mind that Euclid had no Algebra; it did not exist at the time.

Proposition 31. Thales' Theorem, circa 600 B.C. An angle inscribed in a semicircle is a right angle.

Proof. Consider triangle $\triangle ABC$, where $m\angle ABC = 90°$. Let $\alpha = m\angle AOB$ and $\beta = m\angle OCB$. We know that $OA = OB = OC$, since they are radii of $\odot O$. Thus, $\triangle AOB$ and $\triangle BOC$ are isosceles. By the Base Angle Theorem, we see that $m\angle ABO = \alpha$ and $m\angle BOC = \beta$.



Since the sum of the measures of the angles in a triangle is 180°, we compute that $m\angle AOB = 180 - 2\alpha$, and $m\angle BOC = 180 - 2\beta$. Since $\angle AOB$ and $\angle BOC$ is a linear pair, there measure sum to 180°; thus

$$180° = m\angle AOB + m\angle BOC = (180 - 2\alpha) + (180 - 2\beta), \text{ so } 2\alpha + 2\beta = 180°, \text{ so } \alpha + \beta = 180°.$$

It is now clear that

$$m\angle ABC = m\angle ABO + m\angle BOC = \alpha + \beta = 90°.$$

□